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كلية الهندسة

قسمي الهندسة الميكانيكية والهندسة الكيمياوية

الميكانيك الهندسي - ستاتك

# الفصل الاول

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# University of Tikrit Engineering College Mechanical & Chemical Engineering Department

# Engineering Mechanics Statics Lectures

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## **Chapter One**

## **1-Introduction**

**Engineering Mechanics** : may be defined as a science which describes and predicts the condition of rest or motion of bodies under the action of forces. It is divided into three parts.



علم السكون <u>Statics:</u>

The science which study the bodies under the effect of forces at rest.

## علم الحركة <u>Dynamics</u>: علم الحركة

The science which study the bodies under the effect of forces at motion

1-1 Basic concepts: space, time, mass, force

 space - The geometric region occupied by bodies whose positions are described by linear and angular measurements relative to a coordinate system.
 1. الفضاء - المنطقة الهندسية التي تتخذها الأجسام والتي توصف مواقعها من خلال القياسات الخطية والزاوي نسبة إلى نظام الإحداثيات.



- 2) Time The measure of the succession of events
- 3) **Mass** The measure of the inertia of a body, which is its resistance to a change of motion.
- 4) Force The external or internal action on the bodies or between bodies.
   Newton developed the fundamentals of mechanics. The concepts above, space, time, and mass are absolute, independent of each other in Newtonian Mechanics.

#### 1-2 Newton's 3 Fundamental Laws

1<sup>st</sup> Law - A particle remains at rest or continues to move in a straight line with a constant speed if there is no unbalanced force acting on it (resultant

force = 0).





3<sup>rd</sup> Law the forces of action and reaction between interacting bodies are equal in
magnitude, opposite in direction, and act along the same line of action (Collinear).



## **1-3 System of Units**

Base units are units of length, mass and time.

	Length	Mass	<u>Time</u>
SI Units	Meter (m)	Kilogram (kg)	Second (s)
English Units	Foot (ft)	Slug (slug)	Second (s)

Force: Newton (N)

Weight is a force. The weight of 1 kg Mass is:

W = m g  $W = (1 \text{ kg})(9.81 \text{ m/ s}^2)$ W = 9.81 N

Get to know SI prefixes.

Units of Area and Volume:

Area has units (Length)<sup>2</sup> Volume has units (Length)<sup>3</sup>

W = mg

m = W/g

Conversion of Units:

1 ft = 12 in. (inches). 5280 ft = 1 mi (mile). 1000 lb = 1 kip (kilo-pound). 2000 lb = 1 ton.

Prefixes:

When a numerical quantity is either very large or very small, the units used to define its size may be modified by using a prefix. Some of the prefixes used in the SI system are shown in Table (1). Each represents a multiple or submultiples of a unit which, if applied successively, moves the decimal point of a numerical quantity to every third place. *(The kilogram is the only base unit that is defined with a prefix)* 

#### For example: $4\ 000\ 000\ N = 4\ 000\ kN\ (kilo-newton) = 4\ MN\ (mega-newton).$ $0.005\ m = 5\ mm\ (milli-meter).$

Notice that the SI system does not include the multiple deca (10) or the submultiple centi (0.01), which form part of the metric system. Except for some volume and area measurements, the use of these prefixes is to be avoided in science and engineering.

#### Table (1)

SI Unit Prefixes					
Multiplication Factor	Prefix	Symbol			
$1\ 000\ 000\ 000\ 000 = 10^{12}$	tera	Т			
$1\ 000\ 000\ 000 = 10^9$	giga	G			
$1\ 000\ 000 = 10^6$	mega	$\mathbf{M}$			
$1\ 000 = 10^3$	kilo	k			
$100 = 10^2$	hecto	h			
10 = 10	deka	da			
$0.1 = 10^{-1}$	deci	d			
$0.01 = 10^{-2}$	centi	с			
$0.001 = 10^{-3}$	milli	m			
$0.000\ 001 = 10^{-6}$	micro	μ			
$0.000\ 000\ 001 = 10^{-9}$	nano	n			
$0.000\ 000\ 000\ 001 = 10^{-12}$	pico	р			

## 1-4 Scalars and Vectors

- Vector quantities : are the quantities which have magnitude and direction .such as: Force , weight , speed , displacement , acceleration ,velocity .
- Scalar quantities : are the quantities which have only magnitude , such as :Time , size , density , volume .

**Representation of vector** 

Bold  $\mathbf{R}$  – Word Processors  $\leftarrow$  Book uses this. Arrow R – Long Hand, Word Processors Underline  $\underline{\mathbf{R}}$  – Long Hand, Typewriter, Word Processors Magnitude of a Vector |R|Book uses italics for all scalars

1-5 Types of Vectors:- There are three types of a vector

1-Free Vector - is vector which may be freely moved creating couples

in space.

2-Sliding Vector - forces action on a rigid body are represented by

vectors which may move or slid along their line of action.

*3-Bound Vector or Fixed Vector* - can not be moved without modifying the conditions of the problem

For 2 vectors to be equal they must have the same:

1). Magnitude

2). Direction

They do <u>not</u> need to have the same point of application. A negative vector of a given vector has same magnitude but opposite direction.

**P** and  $-\mathbf{P}$  are equal and opposite  $\mathbf{P} + (-\mathbf{P}) = 0$ 

Resolution of vector into components A single vector can be represented by 2 or more vectors. These vectors are called <u>components</u> of the original vector. Finding these is called resolving the vector into its components.

There is an infinite number of ways to resolve one vector.



(a)

**K**2 1-<sup>v</sup>simple example.



What are the x and y components of **P** if P = 1000 N, and  $\theta = 30^{\circ}$ ?

 $P_x = P \cos 30^\circ = 866 \text{ N}$   $P_y = P \sin 30^\circ = 500 \text{ N}$ 

Note: Given  $P_x$  and  $P_y$ , what is P?

 $P^2 = P_x^2 + P_y^2 = 866^2 + 500^2 = 1000 \text{ N} \dots$  From where???

## 1.7 – Types of Force systems

## 1 - Concurrent Force Systems أنظمة القوى الملتقية في نقطة واحدة

A concurrent force system contains forces whose lines-of action are meeting or intersect at one point. Forces may be *tensile (pulling)* 



Forces may be *compressive (pushing*)



Force exerted on a body has two effects:

- The *external effect*, which is effect to change the state of that body from rest to motion or vise versa.
- The *internal effect*, which is effect between the particles or against the external effect.

If the forces system acting on a body produces no external effect(motion or rotation), the forces are said to be in *balance* and the body states no change in motion is said to be in *equilibrium*.

*2- Collinear* forces : قوى تلتقي في نقطة واحدة او قوى تلتقي خطوط افعالها في نقطة واحدة
 If several forces are intersecting at same point, they are said to be collinear.
 (i.e their lines of actions intersecting in one point)

## قوى في مستوى واحد : 3- Coplanar forces

When all forces acting on a body are in the same plane, the forces are coplanar.

## **1.8 - Principle of Transmissibility**( on line of action)

مبدءا نقل القوة على خط فعلها

The principle of transmissibility states that the condition of equilibrium or of motion of a rigid body will remain unchanged if a *force F* action at a given point of the rigid body is replace by a *force F*' of the same magnitude and the same

direction, but acting at a different point, provided that the two forces have the same line of action.



1.8-

**Resultant Forces** محصلة القوى R:resultant, Rx: X-axie resultant component  $R = R_x + R_y$ where R<sub>v</sub>: Y-axie resultant component R<sub>x</sub> (all forces & components in X-direction) And R<sub>v</sub> (all forces & components in Y-direction)

## **\*\*\*Coordinates types**\*\*\*

There are two types of coordinates A - two dim. coordinates (perpendicular & non perpen....). B - three dim. coordinates.

If two forces  $F_1$  and  $F_2$  acting on a particle A may be replaced by a single force R, which has the same effect on the particle This force is called the Resultant of the forces  $F_1$  and  $F_2$  and may be obtained by constructing

1- A parallelogram, using  $F_1$  and  $F_2$  as two sides of the parallelogram. The diagonal that pass through A represents the resultant. This is known as the *parallelogram law* for the addition of two forces.



2- Redraw a half portion of the parallelogram to illustrate the triangular head - to - tail addition of the components.



3- From this

resultant force

triangle. the magnitude of the can be determined using the

law of cosines. and its direction is determined from the law of sins. The magnitudes of two forces components are determined from the law of sines.

Law of sine and law of cosines (two forces) can see in figure below



## Example 1:-

The screw eye in Fig. 1 is subjected to two forces  $F_1$  and  $F_2$ . Determine the magnitude and direction of the resultant force.



*الحل*:-نرسم متوازي الأضلاع الذي تشكل القوتان ضلعين متجاورين فيه وبالاعتماد على اتجاه القوتين بالنسبة نرسم متوازي الأضلاع الذي تشكل القوتان ضلعين متجاورين فيه وبالاعتماد على اتجاه القوتين بالنسبة **Parallelogram Law**. The parallelogram is formed by drawing a line from the head of  $\mathbf{F}_1$  that is parallel to  $\mathbf{F}_2$  and another line from the head of  $\mathbf{F}_2$ . That is parallel to  $\mathbf{F}_1$  The resultant force  $\mathbf{F}_R$  extends to where these lines intersect at point A. Fig. 1- 1. The two unknowns are the magnitude of  $\mathbf{F}_R$  and the angle  $\boldsymbol{\theta}$ (theta).



نأخذ احد المثلثين الذين يكونان متوازي الأضلاع ونطبق علية قوانين المثلثات قانون الجيب و قانون الجيب تمام .(وهذين القانونين يطبقان فقط في حالة المحاور الغير متعامدة (قانوني الجيب والجيب تمام))



**Trigonometry**. From the parallelogram. the vector triangle is constructed. Figure above . Using the law of Cosines.

 $F_{R} = \sqrt{(100 \text{ N})^{2} + (150 \text{ N})^{2} - 2(100 \text{ N})(150 \text{ N}) \cos 115^{\circ}}$   $= \sqrt{10 000 + 22 500 - 30 000(-0.4226)} = 212.6 \text{ N}$  = 213 NApplying the law of sines to determine  $\theta$ ,  $\frac{150 \text{ N}}{\sin \theta} = \frac{212.6 \text{ N}}{\sin 115^{\circ}} \qquad \sin \theta = \frac{150 \text{ N}}{212.6 \text{ N}} (\sin 115^{\circ})$   $\theta = 39.8^{\circ}$ Thus, the direction  $\phi$  (phi) of  $\mathbf{F}_{R}$ , measured from the horizontal, is  $\phi = 39.8^{\circ} + 15.0^{\circ} = 54.8^{\circ}$ Available of the second of th

- الطريقة التحليلية (يفضل استخدام هذه الطريقة ليجاد محصلة مجموعة من المتجهات في مستوى 2 واحد وتستخدم في حالة المحاور المتعامدة فقط)

2- Two Dimensional force system Resultants

The vector components are written as

$$\vec{F} = F_x \vec{i} + F_y \vec{j}$$



where,

i - the unit vector in the x direction

In twordimensioneraiforce can be described using a magnitude |F| and single angle,

 $\boldsymbol{\theta}$ . The components of the vector are  $F_x$  and  $F_y$ 

$$|\mathbf{F}| = \sqrt{F_x^2 + F_y^2}$$
$$\theta = \tan^{-1} \left(\frac{F_y}{F_x}\right) \Longrightarrow \begin{cases} F_x = \mathbf{F} \cos \theta \\ F_y = \mathbf{F} \sin \theta \end{cases}$$

• Addition of Several Forces.

If more than two forces are to be added. *each force is first resolved into its x and y components and then the respective components are added using scalar algebraic since they arc collinear. The resultant force is then formed by adding the resultant components as shown below* 



Using Cartesian is first represented as a Cartesian vector, i.e,.

The vector resultant is  $F_{R} = (F_{Rx})i + (F_{Ry})j$   $F_{Rx} = \sum F_{x}$   $F_{Rx} = F_{1x} - F_{2x} + F_{3x}$   $F_{Ry} = \sum F_{y}$   $F_{Ry} = F_{1y} + F_{2y} - F_{3y}$   $F_{R} = (F_{1x} - F_{2x} + F_{3x})I + (F_{1y} + F_{2y} - F_{3y})j$   $F_{2y}$   $F_{1y}$   $F_{1y}$   $F_{1s}$   $F_{3y}$ 

vector notation each force

The magnitude of  $F_R$  is then found from the Pythagorean theorem: that is.

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

Also, the angle  $\theta$ , which specifics the direction of the resultant force, is determined from trigonometry.



## Example 2.

Determine the x and y components of F1 and F2 acting on the boom shown in Fig. below . Express each force as a Cartesian vector.



## **Solutions:-**

 $F_1$ ,  $F_2$  are resolved into x and y components.

 $F_{1x} = -200 \text{ Sin } 30^{\circ} \text{ N} = -100 \text{ N} = 100 \text{ N} \checkmark$   $F_{1y} = 200 \text{ Cos } 30 \text{ N} = 173 \text{ N}$   $F_{2x} = 260 \text{ N}(12/13) = 240 \text{ N}$ Similarly  $F_{2y} = 260 \text{ N}(5/13) = 100 \text{ N}$ 



Cartesian Vector Notation. Having determined the magnitude

and directions of the components of each force. we can express each force as a Cartesian vector.

 $F_1 = \{-100i + 173j\} N$  $F_2 = \{ 240 i - 100 j\} N$ 

## **Important points**

• The resultant of several coplanar forces can easily be determined if an x , y coordinate system is established and the forces arc resolved along the axes.

• The direction of each force is specified by the angle its line of action makes with one of the axes, or by a sloped triangle.

• The orientation of the x and y axes is arbitrary and their positive direction can be specified by the Cartesian unit vectors i and j.

• The x and y components of the resultant forces 'arc simply the algebraic addition of the components of all the coplanar forces.

• The magnitude of the resultant force is determined from the Pythagorean theorem, and when the components arc sketched *on the x and y axes, the direction can be determined from trigonometry.* 

*Example 1-1* :- The fixed structure shown below.

P = 500 NT = 200 N

Find: Combine P and T into a single force R



## **Solution :-**

$$\tan \alpha = \frac{BD}{AD} = \frac{5\sin 75}{3+5\cos 75}$$
$$\alpha = 48.4$$

Law of cosines:  $c^{2} = a^{2} + b^{2} - 2ab\cos(c)$   $R^{2} = 200^{2} + 500^{2} - 2(200)(500)\cos(48.4)$ R = 396.5 N



Law of sines:

 $\frac{200}{\sin\theta} = \frac{396.5}{\sin 48.4} \qquad \qquad \theta = 22.2$ 

R = 396.5 N 22.2

**Example 1-2**: A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is 5000 lbf directed along the axis of the barge, determine

- a) the tension in each of the ropes for  $\alpha = 45^{\circ}$ ,
- b) the value of  $\alpha$  for which the tension in rope 2 is a minimum.



Solution :-

- Find a trigonometric solution by applying the Triangle Rule for vector addition. With the magnitude and direction of the resultant known and the directions of the other two sides parallel to the ropes given, apply the Law of Sines to find the rope tensions.
- Trigonometric solution Triangle Rule with Law of Sines







*Example 1- 3* :- The vertical force F of 350 lbs acts downward at A on the two - member frame.



Find: The magnitudes of the two components of **F** directed along AB and AC.



 $\frac{F_{AB}}{\sin 60} = \frac{F_{AC}}{\sin 45} = \frac{350}{\sin 75}$  $F_{AB} = 314 \text{ lbs}$  $F_{AC} = 256 \text{ lbs}$ 

#### Example 1-4

Determine the magnitude and direction measured counterclockwise from the positive x axis of the resultant force of the three forces acting on the ring A. Take  $F_1 = 500$  N and  $\theta = 20^{\circ}$ .



The problem has three forces look at the sum of the forces in the x and y direction

$$\begin{split} F_{R} &= \sum F \\ F_{Rx} &= \sum F_{x} = -600 \text{ N} \left(\frac{4}{5}\right) + 500 \text{ N} \left(\sin\left(20^{\circ}\right)\right) + 400 \text{ N} \left(\cos\left(30^{\circ}\right)\right) \\ &= 37.42 \text{ N} \\ F_{Ry} &= \sum F_{x} = 600 \text{ N} \left(\frac{3}{5}\right) + 500 \text{ N} \left(\cos\left(20^{\circ}\right)\right) + 400 \text{ N} \left(\sin\left(30^{\circ}\right)\right) \\ &= 1029.85 \text{ N} \end{split}$$

The resultant is

$$|F_{\rm R}| = \sqrt{(37.42 \text{ N})^2 + (1029.85 \text{ N})^2} = 1030.5 \text{ N}$$
$$\tan\theta = \frac{F_{\rm Ry}}{F_{\rm Rx}} \Longrightarrow \theta = \tan^{-1}\left(\frac{1029.85 \text{ N}}{37.42 \text{ N}}\right) = 87.92^0$$

*Example 1-5* Determine the x and y components of each force acting on the gusset plate of the bridge truss. Show that the resultant force is zero.



The problem has fours forces look at the sum of the forces in the x and y direction

$$\begin{split} F_{R} &= \sum F \\ F_{Rx} &= \sum F_{x} = F_{1x} + F_{2x} + F_{3x} + F_{4x} \\ &= -200 \ lb \ + 400 \ lb \bigg( \frac{4}{5} \bigg) + 300 \ lb \bigg( \frac{3}{5} \bigg) - 300 \ lb \\ &= 0 \ lb \\ F_{Ry} &= \sum F_{y} = F_{1y} + F_{2y} + F_{3y} + F_{4y} \\ &= 0 \ lb \ + 400 \ lb \bigg( -\frac{3}{5} \bigg) + 300 \ lb \bigg( \frac{4}{5} \bigg) - 0 \ lb \\ &= 0 \ lb \end{split}$$

*Example 1-6* Express each of the three forces acting on the column in Cartesian vector form and compute the magnitude of the resultant force.



Solution :- The problem has three forces look at the sum of the forces in the x and y direction

$$\begin{split} F_{1} &= 150 \ \text{lb}\left(\frac{3}{5}\right)\hat{i} - 150 \ \text{lb}\left(\frac{4}{5}\right)\hat{j} = 90 \ \text{lb} \ \hat{i} - 120 \ \text{lb} \hat{j} \\ F_{2} &= -275 \ \text{lb} \hat{j} \\ F_{3} &= -75 \ \text{lb}\left(\cos\left(60^{\circ}\right)\right)\hat{i} - 75 \ \text{lb}\left(\sin\left(60^{\circ}\right)\right)\hat{j} = -37.5 \ \text{lb} \ \hat{i} - 64.95 \ \text{lb} \ \hat{j} \\ F_{R} &= \sum F \\ F_{Rx} &= \sum F_{x} = F_{1x} + F_{2x} + F_{3x} \\ &= 90 \ \text{lb} + 0 \ \text{lb} - 37.5 \ \text{lb} \\ &= 52.5 \ \text{lb} \\ F_{Ry} &= \sum F_{y} = F_{1y} + F_{2y} + F_{3y} \\ &= -120 \ \text{lb} - 275 \ \text{lb} - 64.95 \ \text{lb} \\ &= -459.95 \ \text{lb} \end{split}$$

The magnitude of the resultant force is

$$|F_{\rm R}| = \sqrt{(52.5 \text{ lb})^2 + (-459.95 \text{ lb})^2} = 462.94 \text{ lb}$$
$$\tan\theta = \frac{F_{\rm Ry}}{F_{\rm Rx}} \Longrightarrow \theta = \tan^{-1} \left(\frac{-459.95 \text{ lb}}{52.5 \text{ lb}}\right) = -83.49^{\circ}$$

*Example 1-7* Determine the magnitude force F so that the resultant  $F_R$  of the three forces is as small as possible. What is the minimum magnitude of  $F_R$ .



Solution :- The problem has three forces look at the sum of the forces in the x and y direction

$$\begin{split} F_{R} &= \sum F \\ F_{Rx} &= \sum F_{x} = F_{1x} + F_{2x} + F_{3x} \\ &= 0 \text{ kN } - F\left(\sin\left(30^{\circ}\right)\right) + 5 \text{ kN} \\ &= 5 \text{ kN} - 0.5F \\ F_{Ry} &= \sum F_{y} = F_{1y} + F_{2y} + F_{3y} \\ &= -4 \text{ kN } + F\left(\cos\left(30^{\circ}\right)\right) + 0 \text{ kN} \\ &= 0.866F - 4 \text{ kN} \end{split}$$

The magnitude squared is equal to

 $|F_{R}|^{2} = (5 \text{ kN} - 0.5\text{F})^{2} + (0.866\text{F} - 4 \text{ kN})^{2}$ 

To find the minimum, we need to take the derivative of the magnitude and set it equal to zero. Take the derivative with respect to F so

$$2F_{R} \frac{dF_{R}}{dF} = 2(5 \text{ kN} - 0.5\text{F})(-0.5) + 2(0.866\text{F} - 4 \text{ kN})(0.866) = 0$$
$$= 2(-2.5 \text{ kN} + 0.25\text{F} + 0.75\text{F} - 3.46 \text{ kN}) = 0$$
$$\Rightarrow F = 5.96 \text{ kN}$$

The magnitude of the resultant force is

$$|F_{R}| = \sqrt{(5 \text{ kN} - 0.5(5.96 \text{ kN}))^{2} + (0.866(5.96 \text{ kN}) - 4 \text{ kN})^{2}}$$
$$= \sqrt{(2.02 \text{ kN})^{2} + (1.16 \text{ kN})^{2}}$$
$$= 2.33 \text{ kN}$$

## Problems .(H.W)

**Q.1-** Four forces act on bolt *A* as shown. Determine the resultant of the force on the bolt.

SOLUTION:

• Resolve each force into rectangular components.



• Calculate the magnitude and direction of the resultant.

1-

force	mag	x-comp	y-comp
$F_1$	150	+129.9	+75.0
$F_2$	80	-27.4	+75.2
$F_3$	110	0	-110.0
$F_4$	100	+96.6	-25.9



 $\boldsymbol{y}$ 

 $20^{\circ}$ 

30°

\$15°

 $F_1 = 150 N$ 

 $F_4 = 100 \text{ N}$ 

x

 $F_2 = 80 N$ 

- Determine the components of the resultant by adding the corresponding force components.
- Calculate the magnitude and direction.
- Calculate the magnitude and direction.

$$R = \sqrt{199.1^2 + 14.3^2} \qquad R = 199.6 \text{N}$$
$$\tan \alpha = \frac{14.3 \text{ N}}{199.1 \text{ N}} \qquad \alpha = 4.1^{\circ}$$

:-



**Q.2-** If  $\phi = 30^{\circ}$  and F2 = 3 kN ,determine the magnitude of the resultant force acting on the plate and its direction measured clockwise from the positive x axis.



**Q.3-** If =  $30^{\circ}$ If and F1 = 250 lb, determine the magnitude of the resultant force acting on the bracket and its direction measured clockwise from the positive x axis.



**Q.4** - The plate is subjected to the two forces at A and B as shown. If  $\theta = 60^{\circ}$ . determine the magnitude of the resultant of these two forces and its direction measured clockwise from the horizontal.



**Q.5-** Three cables pull on the pipe such that they create a resultant force having a magnitude of 900 lb. if two of the cables are subjected to known forces, as shown in the figure. Determine the angle  $\theta$  of the third cable so that the magnitude of force in the cable is a minimum. All forces lie in the x-y plane. What is the magnitude of F?



**Q.6** - If  $\theta = 30^{\circ}$  and T = 6 kN, determine the magnitude of the resultant force acting on the eyebolt and its direction measured clockwise from the positive x axis.



**Q.7-**Determine the magnitude of the resultant force acting on the bracket and its direction measured counterclockwise from the positive u axis.

